

Data Input Form

Default input values are shown below

Cosmology Model	
Enter values - then click button	
<input type="text" value="1091.0"/>	CMB_z (Now)
<input type="text" value="69.6"/>	H_0
<input type="text" value="0.286"/>	Ω_M
<input type="text" value="3.0"/>	z (redshift)
<input type="button" value="Open"/>	<input type="button" value="Flat"/>
<input type="text" value="0.714"/>	Ω_{vac}
<input type="button" value="Manual <math>\Omega_r</math>"/>	<input type="button" value="General"/>
Alternate Display Modes	
<input type="button" value="View at <math>z</math>"/>	<input type="button" value="Show <math>z</math> Tables"/>
<input type="text" value="No Notes"/>	
<input type="button" value="Documentation PDF"/>	
Open Button Sets $\Omega_{vac} = 0$ thus, giving an open Universe. [if the entered $\Omega_M < 1$]	
Flat Button (Recommended) Sets $\Omega_{vac} = 1 - \Omega_M - \Omega_R$ thus, giving a flat Universe.	
General Button (default) Uses Ω_{vac} entered. Ω_R is set to $4.165E-5 / (H_0/100)^2$ Ω_K is set to $\Omega_K = 1 - \Omega_M - \Omega_{vac} - \Omega_R$	

Detailed Input Data

The input data form shown is located on the left side of the output page.

H_0 (Now)

This input item refers to the redshift (**z value**) of the **farthest light currently visible from Earth today** (the Observer's position) and is known as the "**Cosmic Microwave Background**" or **CMB** and defines the edge of the Visible Universe.

This value is somewhere between 1089 and 1100 depending on which study you are reading. The COBE mission came in about 1089; WMAP at around 1091; and Planck at ≈ 1100 . The difference is very small between these values as it relates to times and distances today.

H_0

This input item refers to the current value of the Hubble Parameter – also known as the **Hubble Constant**. The Hubble Constant is the expansion rate of the universe expressed in kilometers (km) per second (s) per megaparsec (Mpc) or "km/s/Mpc."

A single parsec is a very long distance – about 3.26 light years – or slightly more than 19,000,000,000,000 (19 trillion) miles. A megaparsec is 1,000,000 parsecs.

Parsec information: <https://en.wikipedia.org/wiki/Parsec>

This is a very important input item. Virtually every calculation requires knowledge of how fast space is expanding at a specific time and that expansion rate is calculated based, in part, upon this input item.

Detailed Input Data (continued-1)

Once again, depending on which study you are reading, **this value is expected to be around 70.0 with error bars of +/- 4.0.**

See: https://en.wikipedia.org/wiki/Hubble%27s_law for detailed information.

The current debate regarding the **Hubble Constant (H₀)** revolves around nearby measurements of light-emitting objects (stars, galaxies, etc.) that seem to yield **approximately H₀ = 74.0 (Riess, et al.)** versus longer-range CMB measurements that seem to yield **approximately H₀ = 67.4 (Planck Team)**.

There are certain gating values – such as the Age of the Universe – that must be met within reasonable bounds for Input Data to be valid. **In the case of the Age of the Universe, this would be 13.750 Gyr +/- 0.100 Gyr.**

To achieve this age range, **Matter Density (Omega M or Ωm – discussed below)** must be adjusted. Shown below is a comparison chart of different Hubble Constants and the corresponding Matter Density Fraction that is required in order to achieve an age of approximately 13.750 Gyr.

Comparison of Hubble Constant vs. Matter Density for age = 13.750 Gyr					
			Geometry:	Flat	
			CMBz =	1091	
Hubble Constant	Omega(m) Matter Density	AGE Now	Age at Last Scattering	Comoving Radial Distance	Last Scattering Proper Distance
(k/s/Mpc)	(Fraction)	(Gyr)	(years)	(Gly)	(ly)
67.400	0.3183	13.750348690	369,632	45.021248040	41,228,249
70.000	0.2779	13.750718605	376,698	45.768088901	41,912,169
74.000	0.2265	13.750559673	387,665	46.951517348	42,995,895

This table shows that various Hubble Constants (67.4, 70.0, and 74.0) can produce the same age (approximately 13.750 Gyr) by simply altering the Matter Density Parameter

Detailed Input Data (continued-2)

Ω_{M}

This input item is the **Matter Density Parameter (Omega M** or, using the Greek letter for omega (Ω), Ω_{M}) corresponding to today. This **Ω_{M} value is input as a fraction** and represents the **sum of normal (baryonic) matter and dark matter**. This fraction changes over time as the universe expands.

See: https://en.wikipedia.org/wiki/Lambda-CDM_model for detailed information.

Ω_{vac}

This input item is the **Dark Energy Density Parameter (Omega vac or Ω_{vac})** corresponding to today. This **Ω_{vac} value is input as a fraction**. This fraction changes over time as the universe expands.

z (redshift)

This input item is the **z value** the model will use for the detailed output of values corresponding to the time (age) of, and related distances for, this z value.

The **bottom half of the default output display** shows values that are associated with or related to **this input z value**.

The **top half of the default output display** shows values that define **the visible universe today** as reflected from the **CMB_z z value** input data discussed above.

Display Options for Output

This model is based upon Dr. Edward (Ned) L. Wright's work and many of the calculations may be found at:

<http://iopscience.iop.org/article/10.1086/510102>

There are certain parameters within the model for which assumptions are made as to their values today. Depending upon which output display option is chosen, these values will adjust accordingly.

The first parameter is the **Radiation Density Parameter** or **Omega r (Ω_r)**. Quoting Dr. Wright's paper:

"Another hidden aspect of the cosmology calculator is that it automatically sets the radiation density $\Omega_r h^2$ to the value appropriate for $T_0 = 2.72528$ K and three massless neutrino species, $\Omega_r h^2 = 4.165 \times 10^{-5}$. Here $h = H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, and this factor includes a small (<1%) boost in the neutrino density due to a slight transfer to e^+, e^- annihilation energy into neutrinos (Hannestad & Madsen 1995)."

Using the default **H_0 of 69.6**, the value for **Ω_r** would be **0.00008598**.

The **Curvature Parameter (Ω_k)** is calculated by the following formula:

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

This assumes the **Total Density Parameter is 1.000** and the Curvature Parameter **Ω_k** is adjusted to the necessary value to achieve this value. Depending upon the choice of output, these parameters are handled in differing manners. Each will be explained below for each output option.

Using the default input fields yields a **Curvature Parameter** value as follows:

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

$$\Omega_k = 1 - 0.00008598 - 0.286 - 0.714$$

$$\Omega_k = -0.00008598$$

Output Options

There are three basic and three more advanced options for the output display controlled by which button the user clicks. The output choice can dictate how the calculation of the Omega values is handled.

It is strongly recommended that nonscientist users choose the Flat geometry display option which will properly set Ω_{vac} .



“Open” Output Option (Basic)

Choosing the **Open** Button causes Ω_{vac} to be set to zero.

The default settings for **Open** are then set to:

$$\text{CMBz} = 1091$$

$$H_0 = 69.6$$

$$\Omega_m = 0.28600000$$

$$\Omega_{vac} = 0.00000000$$

$$\Omega_r = 0.00008598$$

$$\Omega_k = 0.71391402$$

“Flat” Output Option (Basic)

Choosing the **Flat** Button causes Ω_k to be set to zero. Ω_{vac} is calculated by:

$$\Omega_{vac} = 1 - \Omega_m - \Omega_r$$

The default settings for **Flat** are then set to:

$$\text{CMBz} = 1091$$

$$H_0 = 69.6$$

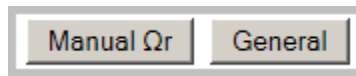
$$\Omega_m = 0.28600000$$

$$\Omega_{vac} = 0.71391402$$

$$\Omega_r = 0.00008598$$

$$\Omega_k = 0.00000000$$

Output Options (continued-1)



“General” Output Option (Basic)

Choosing the **General** Button causes Ω_k to be calculated by:

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

The default settings for **General** are then set to:

$$\text{CMBz} = 1091$$

$$H_o = 69.6$$

$$\Omega_m = 0.28600000$$

$$\Omega_{vac} = 0.7140000$$

$$\Omega_r = 0.00008598$$

$$\Omega_k = -0.00008598$$

“Manual Ωr” Output Option (advanced)

Choosing the **Manual Ωr** Button causes an input box to pop-up and allows to user to manually enter the Ω_r value. Again, Ω_k will be calculated by:

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

The default settings for **Manual Ωr** are then set to:

$$\text{CMBz} = 1091$$

$$H_o = 69.6$$

$$\Omega_m = 0.28600000$$

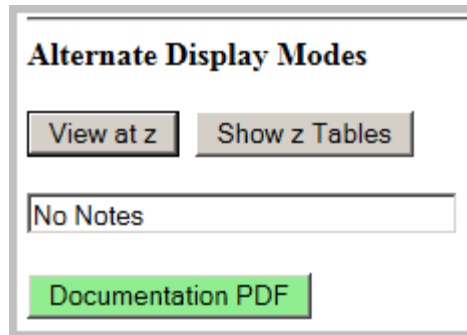
$$\Omega_{vac} = 0.7140000$$

$$\Omega_r = \text{[What user enters in Pop-up box]}$$

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

Output Options (continued-2)

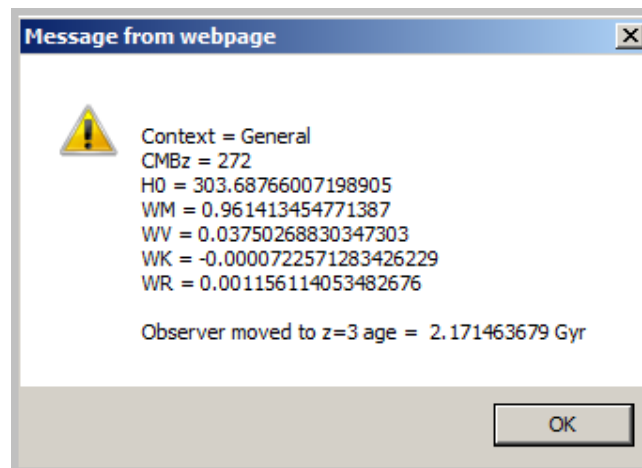
Note: If any input data field is changed, you must select the OPEN, FLAT, MANUAL Ω_r , or GENERAL Button before choosing any Advanced Display Mode.



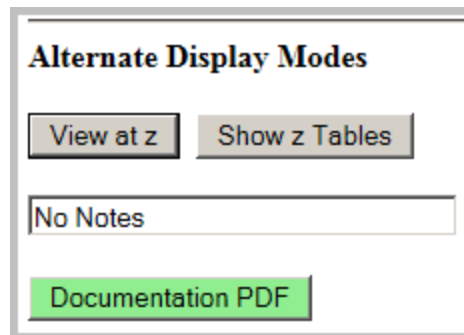
“View at z” Output Option (Advanced)

Choosing the **View at z** Button causes the “Observer” to move to the age of the previously-calculated z value and using the same output option choice (Open, Flat, General, Manual Ω_r) that was last used.

For example, using the default input values and the “**General**” output option, the age calculated for the default z=3.000 setting is 2.171463679 Gyr. Clicking the **View at z** button moves the Observer and all settings to that age. The Input Data Form items are automatically repopulated by the model as shown below:



Output Options (continued-3)



Alternate Display Modes

View at z Show z Tables

No Notes

Documentation PDF

“Show z Tables” Output Option (Advanced)

Choosing the **Show z Tables** Button displays many pages of different calculations at **various z values from 0.000 to CMBz** (default CMBz = 1091).

These tables have detailed calculations that allow the user to see trends and further show special z values, including the **Maximum LS-Photon proper distance z** and the **universe-begins-accelerating z**.

The **z Tables** will be calculated using the same output option choice (**Open, Flat, General, or Manual Ω_r**) that was last used.

There are two pages of the eight z Tables shown below in this document. These two example tables show hoe the Hubble Parameter is calculated for a given z point value.

Use of the Omega Values

The reason so much emphasis regarding the Input Data Form is put upon properly setting the **Omega Values** is that the calculation of the changing **Hubble Parameter** that defines the expansion rate of the universe at any point in time is governed by a combination of the current **Hubble Constant (H₀)**, **the Scale Value z**, and **the Omega Values**. The formula for Hubble Parameter at a given z is:

$$E(z) = \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}$$

$$H(z) = H_0 E(z)$$

For more, see: [https://en.wikipedia.org/wiki/Distance_measures_\(cosmology\)](https://en.wikipedia.org/wiki/Distance_measures_(cosmology))

This model is generally set up to calculate the expansion of the universe from the Last Scattering (approximately 380,000 years after the Big Bang) until today. Or, for advanced users, to calculate beyond today into the future and to calculate the Hubble Parameters at z values prior to the Last Scattering.

It is believed that **Ω_r (radiation)** dominated the first 50,000 years (z=3325) after the Big Bang. From 50,000 years until approximately 9.8 **billion** years, **Ω_m (Matter)** dominated. From then until now, **Ω_{vac} (Dark Energy)** dominated.

This **Ω_m** to **Ω_{vac}** transition can be seen on z Table-3 below. Look between z=0.30 and z=0.40. The **Ω_{vac}** fraction exceeds 0.50 while the **Ω_m** fraction drops below 0.50. The exact z value for this specific Default Input Form data set is close to z=0.355 (which shows the age at approximately 9.8 billion years after the Big Bang).

For more, see: [https://en.wikipedia.org/wiki/Scale_factor_\(cosmology\)](https://en.wikipedia.org/wiki/Scale_factor_(cosmology))

The tables on the next two pages show the calculated **Hubble Parameter** and the **Omega Fractions** at various z values. These two pages are included in the larger set of **z Table** pages that are output with this “**Show z Tables**” option.

Table 2 - For each z --> Shows Hubble at z, OmegaSum, and raw Omega values at z

For $H_0 = 69.600$ $\Omega_{\text{Omega}_m} = 0.28600000$ $\Omega_{\text{vac}} = 0.71400000$ $\Omega_k = -0.00008598$ $\Omega_r = 0.00008598$

[General]Hubble....
.....	..Parameter..E(Q s)...Q m^3....	...Q v^1..	...Q k^2...Q r^4.....
.Redshiftat z.....	...OmegaSum..Value....	...Value..	...Value...Value.....
....(z)..	..(k/s/Mpc)..(at_z)...(at_z)...	..(at_z)..	... (at_z)..(at_z)....
0.00	69.60000	1.00000	0.28600000	0.71400000	-0.00008598	0.00008598
0.01	69.90099	1.00867	0.29466609	0.71400000	-0.00008771	0.00008947
0.02	70.20667	1.01751	0.30350549	0.71400000	-0.00008945	0.00009307
0.03	70.51705	1.02653	0.31251992	0.71400000	-0.00009122	0.00009677
0.04	70.83210	1.03572	0.32171110	0.71400000	-0.00009300	0.00010058
0.05	71.15185	1.04509	0.33108075	0.71400000	-0.00009479	0.00010451
0.06	71.47627	1.05464	0.34063058	0.71400000	-0.00009661	0.00010855
0.07	71.80536	1.06438	0.35036230	0.71400000	-0.00009844	0.00011270
0.08	72.13913	1.07429	0.36027763	0.71400000	-0.00010029	0.00011697
0.09	72.47755	1.08440	0.37037829	0.71400000	-0.00010215	0.00012137
0.10	72.82062	1.09469	0.38066600	0.71400000	-0.00010404	0.00012588
0.20	76.50501	1.20826	0.49420800	0.71400000	-0.00012381	0.00017829
0.30	80.64121	1.34244	0.62834200	0.71400000	-0.00014531	0.00024557
0.40	85.21228	1.49895	0.78478400	0.71400000	-0.00016852	0.00033030
0.50	90.19827	1.67949	0.96525000	0.71400000	-0.00019345	0.00043527
0.60	95.57779	1.88580	1.17145600	0.71400000	-0.00022011	0.00056348
0.70	101.32927	2.11959	1.40511800	0.71400000	-0.00024848	0.00071811
0.7092	101.87635	2.14254	1.42805418	0.71400000	-0.00025118	0.00073378
0.80	107.43174	2.38258	1.66795200	0.71400000	-0.00027857	0.00090258
0.90	113.86535	2.67648	1.96167400	0.71400000	-0.00031039	0.00112050
1.00	120.61163	3.00303	2.28800000	0.71400000	-0.00034392	0.00137568
1.50	158.49207	5.18557	4.46875000	0.71400000	-0.00053737	0.00335859
1.622403	168.70318	5.87527	5.15779803	0.71400000	-0.00059128	0.00406625
2.00	202.22592	8.44219	7.72200000	0.71400000	-0.00077382	0.00696437
2.50	250.83148	12.98810	12.26225000	0.71400000	-0.00105325	0.01290235
3.00	303.68766	19.03864	18.30400000	0.71400000	-0.00137568	0.02201083
3.50	360.37255	26.80927	26.06175000	0.71400000	-0.00174109	0.03525710
4.00	420.57978	36.51559	35.75000000	0.71400000	-0.00214950	0.05373739
4.50	484.07451	48.37333	47.58325000	0.71400000	-0.00260089	0.07867691
5.00	550.66900	62.59833	61.77600000	0.71400000	-0.00309527	0.11142985
5.50	620.20824	79.40660	78.54275000	0.71400000	-0.00363265	0.15347935
6.00	692.56101	99.01422	98.09800000	0.71400000	-0.00421301	0.20643755
6.50	767.61404	121.63746	120.65625000	0.71400000	-0.00483636	0.27204552
7.00	845.26806	147.49267	146.43200000	0.71400000	-0.00550271	0.35217334
7.50	925.43495	176.79636	175.63975000	0.71400000	-0.00621204	0.44882003
8.00	1,008.03569	209.76515	208.49400000	0.71400000	-0.00696437	0.56411359
8.50	1,092.99881	246.61580	245.20925000	0.71400000	-0.00775968	0.70031100
9.00	1,180.25923	287.56520	286.00000000	0.71400000	-0.00859798	0.85979819
9.50	1,269.75727	332.83036	331.08075000	0.71400000	-0.00947928	1.04509007
Decimals drop here due to width limitations						
10.00	1,361.4	383	380.7	0.71400000	-0.010	1.259
25.00	4,954.2	5,067	5,026.7	0.71400000	-0.058	39.291
53.60	15,139.8	47,317	46,552.6	0.71400000	-0.256	764.130
67.25	21,201.0	92,789	90,923.1	0.71400000	-0.400	1,865.551
90.00	32,750.3	221,417	215,521.3	0.71400000	-0.712	5,896.063
135.500	60,565.3	757,232	727,384.4	0.71400000	-1.602	29,848.818
181.000	93,857.0	1,818,505	1,724,170.4	0.71400000	-2.848	94,337.004
272.000	174,648.2	6,296,651	5,819,075.3	0.71400000	-6.408	477,581.082
363.000	272,266.3	15,302,745	13,793,363.6	0.71400000	-11.392	1,509,392.060
545.000	512,370.8	54,193,874	46,552,602.1	0.71400000	-25.632	7,641,297.304
1091.000	1,548,003.9	494,681,472	372,420,816.8	0.71400000	-102.528	122,260,756.865

Table 3 - For each z --> Hubble Components for "View at z"

For $H_0 = 69.600$ $\Omega_{\text{matter}} = 0.28600000$ $\Omega_{\text{vac}} = 0.71400000$ $\Omega_{\text{K}} = -0.00008598$ $\Omega_{\text{R}} = 0.00008598$

[General]	...CMB...	...Hubble...
.....	.Redshift	..Parameter.E (Qs)... Ω_m Ω_v Ω_k Ω_r ...
.Redshift	...at z..at z....SQRT....	.Fraction.	.Fraction.	..Fraction.	.Fraction.
....(z)..(\bar{z})..	..(k/s/Mpc).(at_z)...	..(at_z)..	..(at_z)..	... (at_z)..	..(at_z)..
0.00	1,091.00	69.60000	1.000000	0.28600000	0.71400000	-0.00008598	0.00008598
0.01	1,080.19	69.90099	1.004325	0.29213391	0.70786434	-0.00008695	0.00008870
0.02	1,069.59	70.20667	1.008717	0.29828282	0.70171362	-0.00008791	0.00009147
0.03	1,059.19	70.51705	1.013176	0.30444439	0.69555020	-0.00008886	0.00009427
0.04	1,049.00	70.83210	1.017703	0.31061630	0.68937638	-0.00008979	0.00009712
0.05	1,039.00	71.15185	1.022297	0.31679626	0.68319444	-0.00009070	0.00010000
0.06	1,029.19	71.47627	1.026958	0.32298203	0.67700665	-0.00009160	0.00010292
0.07	1,019.56	71.80536	1.031686	0.32917138	0.67081522	-0.00009248	0.00010589
0.08	1,010.11	72.13913	1.036482	0.33536213	0.66462234	-0.00009335	0.00010889
0.09	1,000.83	72.47755	1.041344	0.34155214	0.65843014	-0.00009420	0.00011192
0.10	991.73	72.82062	1.046273	0.34773931	0.65224073	-0.00009504	0.00011499
0.20	909.00	76.50501	1.099210	0.40902371	0.59093120	-0.00010247	0.00014756
0.30	839.00	80.64121	1.158638	0.46805886	0.53186645	-0.00010824	0.00018293
0.40	779.00	85.21228	1.224314	0.52355730	0.47633477	-0.00011243	0.00022035
0.50	727.00	90.19827	1.295952	0.57472742	0.42512860	-0.00011519	0.00025917
0.60	681.50	95.57779	1.373244	0.62119864	0.37861928	-0.00011672	0.00029880
0.70	641.35	101.32927	1.455880	0.66292046	0.33685798	-0.00011723	0.00033880
0.7092	637.90	101.87635	1.463741	0.66652493	0.33324982	-0.00011723	0.00034248
0.80	605.67	107.43174	1.543560	0.70006245	0.29967564	-0.00011692	0.00037883
0.90	573.74	113.86535	1.635996	0.73292944	0.26676788	-0.00011597	0.00041865
1.00	545.00	120.61163	1.732926	0.76189670	0.23775972	-0.00011452	0.00045810
1.50	435.80	158.49207	2.277185	0.86176620	0.13768975	-0.00010363	0.00064768
1.622403	415.41	168.70318	2.423896	0.87788228	0.12152627	-0.00010064	0.00069210
2.00	363.00	202.22592	2.905545	0.91469151	0.08457521	-0.00009166	0.00082495
2.50	311.00	250.83148	3.603901	0.94411429	0.05497340	-0.00008109	0.00099340
3.00	272.00	303.68766	4.363328	0.96141345	0.03750269	-0.00007226	0.00115611
3.50	241.67	360.37255	5.177767	0.97211725	0.02663258	-0.00006494	0.00131511
4.00	217.40	420.57978	6.042813	0.97903394	0.01955329	-0.00005887	0.00147163
4.50	197.55	484.07451	6.955094	0.98366711	0.01476020	-0.00005377	0.00162645
5.00	181.00	550.66900	7.911911	0.98686332	0.01140605	-0.00004945	0.00178008
5.50	167.00	620.20824	8.911038	0.98912122	0.00899170	-0.00004575	0.00193283
6.00	155.00	692.56101	9.950589	0.99074654	0.00721109	-0.00004255	0.00208493
6.50	144.60	767.61404	11.028937	0.99193333	0.00586990	-0.00003976	0.00223653
7.00	135.50	845.26806	12.144656	0.99280866	0.00484092	-0.00003731	0.00238773
7.50	127.47	925.43495	13.296479	0.99345796	0.00403854	-0.00003514	0.00253863
8.00	120.33	1,008.03569	14.483271	0.99394013	0.00340381	-0.00003320	0.00268926
8.50	113.95	1,092.99881	15.704006	0.99429659	0.00289519	-0.00003146	0.00283968
9.00	108.20	1,180.25923	16.957747	0.99455706	0.00248292	-0.00002990	0.00298992
9.50	103.00	1,269.75727	18.243639	0.99474324	0.00214524	-0.00002848	0.00314001
Decimals drop here due to width limitations							
10.00	98.27	1,361.4	19.560890	0.99487119	0.00186604	-0.00002719	0.00328996
25.00	41.00	4,954.2	71.180634	0.99211583	0.00014092	-0.00001147	0.00775472
53.60	19.00	15,139.8	217.525147	0.98384123	0.00001509	-0.00000542	0.01614909
67.25	15.00	21,201.0	304.612730	0.97989130	0.00000769	-0.00000432	0.02010532
90.00	11.00	32,750.3	470.550073	0.97337126	0.00000322	-0.00000322	0.02662873
135.500	7.00	60,565.3	870.190978	0.96058286	0.00000094	-0.00000212	0.03941831
181.000	5.00	93,857.0	1,348.519676	0.94812505	0.00000039	-0.00000157	0.05187612
272.000	3.00	174,648.2	2,509.312784	0.92415406	0.00000011	-0.00000102	0.07584684
363.000	2.00	272,266.3	3,911.872310	0.90136532	0.00000005	-0.00000074	0.09863538
545.000	1.00	512,370.8	7,361.648897	0.85900118	0.00000001	-0.00000047	0.14099928
1091.000	0.00	1,548,003.9	22,241.435921	0.75284974	0.00000000	-0.00000021	0.24715047

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