Reciprocal Difference Formula

In **mathematics**, a **multiplicative inverse** or **reciprocal** for a number *x*, normally denoted by 1/*x*, is a number which when multiplied by *x* yields 1. The reciprocal of the **fraction** *a*/*b* is *b*/*a*. For the reciprocal of a real number, divide 1 by the number. For example, the reciprocal of 5 is one fifth (1/5 or 0.2), and the reciprocal of 0.25 is 1 divided by 0.25, or 4.

Calculating the difference between a reciprocal pair can be done by subtracting the lower value from the higher value. For example, given a reciprocal pair such as 4 and 0.25, the difference is 4 – 0.25 = 3.75.

Identifying the reciprocal pair values when given just their difference is difficult. For example, using a reciprocal pair difference of 5, the corresponding reciprocal pair is 5.19258240357… and 0.19258240357….

The Reciprocal Difference Formula is:

Reciprocal = SQRT (n^2+4)/2 ± n/2

where,
n = Reciprocal pair difference

Note: The high reciprocal uses add (+) n/2 and the lower reciprocal uses minus (-) n/2.

For example, for a reciprocal pair difference (n) of 5:

High Reciprocal = SQRT(5^2+4)/2 + 2.5 = 5.19258240357…
Low Reciprocal = SQRT(5^2+4)/2 – 2.5 = 0.19258240357…
Difference = 5.00000000000

Reciprocal Difference Formula Derivation

Using n for the difference between a reciprocal pair;

Lemma	n = x – 1/x
Multiply by x	n • x = x^2 – 1
Subtract (n • x)	x^2 – n • x – 1 = 0
Apply Quadratic Formula	x = [-(-n) ± SQRT(n^2 – (4 • 1 • -1)]/2
Simplify & rearrange	**x = SQRT(n^2 + 4)/2 ± n/2**

Why the Reciprocal Difference Formula Works

Excepting the value 1 (and its reciprocal of 1), all positive reciprocal pairs have one value greater than 1 and one value between 0 and 1. There is always a point halfway (midpoint) between the reciprocal pair that is half of the difference (n/2) from the higher and lower value. Hence, the term ± n/2 is used in the Reciprocal Difference Formula.

The rest of the formula calculates the midpoint of the reciprocal pair. For example:

Reciprocal Difference (n)	Midpoint SQRT (n^2+4)/2	High Reciprocal Plus (n/2)	Low Reciprocal Minus(n/2)
	- - - - - - - - - - - - - Decimals to 11 digits	- - - - - - - - - - - - -	
0	1.00000000000	1.00000000000	1.00000000000
1	1.11803398875	1.61803398875	0.61803398875
2	1.41421356237	2.41421356237	0.41421356237
3	1.80277562773	3.30277562773	0.30277562773
4	2.23606797750	4.23606797750	0.23606797750
5	2.69258240357	5.19258240357	0.19258240357
3.75	2.12500000000	4.00000000000	0.25000000000
4.5	2.46221445045	4.71221445045	0.21221445045

Application to the Golden Ratio

The Golden Ratio which is often denoted as the Greek letter phi (φ or ϕ) has a value of:

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887\ldots.$$

The reciprocal of the Golden Ratio is 0.6180339887. The difference (n) for this reciprocal pair is 1.00000000000.

Using the Reciprocal Difference Formula and the Golden Ratio difference of 1:

High Reciprocal = SQRT(1^2+4)/2 + 1/2 = 1.1180339887 + 0.5 = 1.61803398875…
Low Reciprocal = SQRT(1^2+4)/2 – 1/2 = 1.1180339887 – 0.5 = 0.61803398875…

The Golden Ratio is the case for “n=1” for the Reciprocal Difference Formula.

Application to the Fibonacci Number Sequence

The Fibonacci Number Sequence is an example of a limit convergence sequence. The convergence, calculated by dividing each number in the sequence by the previous number in the sequence, oscillates above and below the higher reciprocal of the Golden Ratio reciprocal pair (=1.61803398875…).

By reversing the division, that is, dividing the previous number in the sequence by the next number in the sequence, the oscillation tracks above and below the reciprocal of the Golden Ratio (=0.61803398875…). The Fibonacci Number Sequence adds the two previous numbers to obtain the next number in the sequence.

	- - - - - - - - - - - - - - - Decimals to 11 digits	- - - - - - - - - - - - -	
Fibonacci Sequence	High Reciprocal Oscillation	Low Reciprocal Oscillation	Difference
1	N/A	N/A	N/A
1	1.00000000000	1.00000000000	0.00000000000
2	2.00000000000	0.50000000000	1.50000000000
3	1.50000000000	0.66666666667	0.83333333333
5	1.66666666667	0.60000000000	1.06666666667
8	1.60000000000	0.62500000000	0.97500000000
13	1.62500000000	0.61764705882	1.00961538462
21	1.61538461538	0.61904761905	0.99633699634
34	1.61904761905	0.61764705882	1.00140056022
55	1.61764705882	0.61818181818	0.99946524064
89	1.61818181818	0.61797752809	1.00020429009
144	1.61797752809	0.61805555556	0.99992197253
233	1.61805555556	0.61802575107	1.00002980448
377	1.61802575107	0.61803713528	0.99998861579
610	1.61803713528	0.61803278689	1.00000434839
987	1.61803278689	0.61803444782	0.99999833906
1,597	1.61803444782	0.61803381340	1.00000063442
Actual Reciprocals	1.61803398875	0.61803398875	1.00000000000

The actual reciprocals converge to 11 digits at Fibonacci Number 514,229.

The Fibonacci Number Sequence oscillations will continue getting closer to the correct answer but cannot exactly match due to the involvement of the irrational square root of 5 in the calculation of the Golden Ratio.

General Additive Number Sequence

Next Sequence Number = [Previous(-2) + Previous(-1) • m]

where,
m = Multiplier (aka Reciprocal Difference)

The Fibonacci Number Sequence is the case for "**m=1**" for the General Additive Number Sequence.

Changing to **m=2** or m=**3.5** or **m=5** and so on, causes the General Additive Number Sequence to converge on a reciprocal pair with a difference of 2 or 3.5 or 5 and so on.

An example for m=2 is shown: **M=2**

General Additive Sequence	High Reciprocal Oscillation	Low Reciprocal Oscillation	Difference
	- - - - - - - - - - - - - Decimals to 11 digits - - - - - -- - - - -		
1	N/A	N/A	N/A
1	1.00000000000	1.00000000000	0.00000000000
3	3.00000000000	0.33333333333	2.66666666667
7	2.33333333333	0.42857142857	1.90476190476
17	2.42857142857	0.41176470588	2.01680672269
41	2.41176470588	0.41463414634	1.99713055954
99	2.41463414634	0.41414141414	2.00049273220
239	2.41414141414	0.41422594142	1.99991547272
577	2.41422594142	0.41421143847	2.00001450295
1,393	2.41421143847	0.41421392678	1.99999751170
3,363	2.41421392678	0.41421349985	2.00000042693
8,119	2.41421349985	0.41421357310	1.99999992675
19,601	2.41421357310	0.41421356053	2.00000001257
47,321	2.41421356053	0.41421356269	1.99999999784
Actual Reciprocals	2.41421356237	0.41421356237	2.00000000000

The actual reciprocals for m=2 converge to 11 digits at General Additive Number 1,607,521.

The Fibonacci Number Sequence is the case for "m=1" for the General Additive Number Sequence.

Correct Quadratic Form for Reciprocal Pairs

Using the general Quadratic Form:

$$ax^2 + bx + c = 0$$

"**a**" is set to 1.
"**b**" is always the opposite of the sum of the two roots (Reciprocal Pair).
"**c**" is always the product of the two roots. Since Reciprocal Pairs multiply to equal 1 – then "**c**" is always set to 1.

As an example, using the Golden Ratio (1.61803398875) and its reciprocal (0.61803398875), the opposite of the sum of the roots would be **-2.23606797750**.

$$x^2 - 2.23606797750x + 1 = 0$$

The two roots will be 1.61803398875 and 0.61803398875.

For a Reciprocal Pair with a difference of 2.000 (2.41421356237 and 0.41421356237), the opposite of the sum of the roots would be **-2.82842671274**.

$$x^2 - 2.82842671274x + 1 = 0$$

The two roots will be 2.41421356237 and 0.41421356237.

This technique works the same way for negative reciprocal pairs. Simply change the "**b**" to the positive form and the roots will be negative.

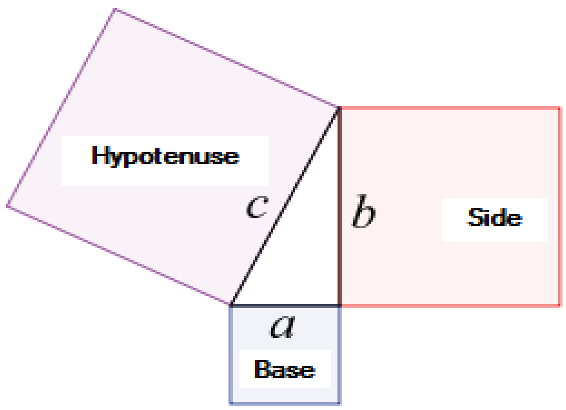
$$x^2 + 2.82842671274x + 1 = 0$$

The two roots will be -2.41421356237 and -0.41421356237.

Correct Pythagorean Triangle Form for Reciprocal Pairs

A Pythagorean Triangle as shown below will also calculate Reciprocal Pairs of a given difference.

Where, *c* ***(Hypotenuse)*** represents the length of the longest side opposite the right angle and with *a* ***(Base)*** and *b* ***(Side)*** representing the lengths of the triangle's other two sides.



Simply set the Base ("**a**") to equal 2. Then set the Side ("**b**") to the Reciprocal Difference.

To calculate the Reciprocal Pair, use the steps below:

Step 1 - calculate $c = \sqrt{a^2 + b^2}.$

Step 2 – High Reciprocal = (**c** + **b)**/2

Step 3 – Low Reciprocal = (**c** – **b)**/2

For example, using a reciprocal difference of 5, **c** = SQRT (2^2 + 5^5) or 5.385164807.

High Reciprocal = (5.385164807 + 5) / 2 = 5.192582404

Low Reciprocal = (5.385164807 – 5) / 2 = 0.192582404

Reciprocal Difference = 5.000000000

Extra Note: While the above example is for Reciprocal Pairs of the form **(x, 1/x)**, the **Base (a) length for any reciprocal form (x, n/x) would equal SQRT (n*4)**.

Reciprocal Pairs of a Given Multiple

The multiple for a reciprocal pair is calculated by dividing the higher reciprocal by the lower reciprocal. For example, the reciprocal pair of 4 and 0.25 would yield a multiple of 16 (4 / 0.25). The formula to generate a specific multiple Reciprocal Pair of a given multiple k is:

Specific Multiple Reciprocal Pair Formula

High Reciprocal Value = SQRT (k)
Low Reciprocal Value = 1 / SQRT (k)

where,
k = Specific reciprocal pair multiple needed

For example, to obtain a Reciprocal Pair with a multiple of 8 (k=8):

High Reciprocal = SQRT (8) = 2.82842712475…
Low Reciprocal = 1 / SQRT (8) = 0.35355339059…
Multiple = 8.00000000000